

Axionic Domain Wall and Warped Geometry

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Abstract

We discuss how a three-brane with an associated non-factorizable (warped) geometry can emerge from a five dimensional theory of gravity coupled to a complex scalar field. The system possesses a discrete Z_2 symmetry, whose spontaneous breaking yields an 'axionic' three-brane and a warped metric. Analytic solutions for the wall profile and warp factor are presented. The Kaluza-Klein decomposition and some related issues are also discussed.

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1 Introduction

Theories with extra spacelike dimensions have recently attracted a great deal of attention. It was observed [1, 2] that for suitably large extra dimensions, it is possible to lower the fundamental mass scale of gravity M_f down to a few TeV. This suggests a new way for a solution of the gauge hierarchy problem without invoking supersymmetry (SUSY). In this approach all the standard model particles are localized on a 3-brane, and only gravity propagates in the bulk. Assuming that the n compact dimensions have a typical size R , the four dimensional Planck scale is expressed as:

$$M_{\text{Pl}}^2 \sim M_f^{2+n} R^n . \quad (1)$$

In order to reproduce the correct behavior of gravity one should take $R \lesssim \text{mm}$ (the behavior of gravity at this distance is now being studied). Interestingly, already for $n = 2$, $R \sim 1 \text{ mm}$ and $M_f \sim \text{few} \cdot \text{TeV}$, M_{Pl} has the required magnitude $\sim 10^{19} \text{ GeV}$. Detailed studies of phenomenological and astrophysical implications of these models, were presented in [2]. We note that our Universe as a membrane embedded in higher dimensional spacetime was also considered in earlier works [3].

An alternative solution of the gauge hierarchy problem invoking an extra dimension was presented in [4]. The desired mass hierarchy is generated through a non-factorizable metric obtained from higher dimensional gravity (see also [5]). In the minimal setting [4] there are two three branes - hidden and visible, separated by an appropriate distance. The non-factorizable metric is given by:

$$ds^2 = e^{-2k|y|} ds_{3+1}^2 - dy^2 , \quad (2)$$

where y denotes the fifth spacelike dimension, ds_{3+1}^2 is the ordinary 4D interval, and k is a mass parameter close to the fundamental scale M_f . On the visible brane all mass parameters are rescaled due to the warp factor in (2), such that $m_{\text{vis}} = M_f e^{-k|y_0|}$ (y_0 is the distance between branes). For $M_f \sim 10^{19} \text{ GeV}$ and $k|y_0| \simeq 37$ one finds that $m_{\text{vis}} \sim \text{few} \cdot \text{TeV}$, the desired magnitude. It was also shown that Newton's law still holds on the visible brane. It is worth noting that in this approach the extra dimension can be infinite [6], provided it's volume remains finite. Generalization of this non-factorizable model to scenarios with open codimensions and with intersecting multiple branes was presented in [7].

It is clearly important to inquire about the origin of the 3-branes in the above scheme with the warped metric. In this paper we present one such scenario with a complex scalar field coupled to 5D gravity. The theory possesses 5D Poincare invariance and Z_2 discrete symmetry. The 3-brane and warped geometry emerge dynamically from spontaneous breaking of the Z_2 symmetry. The 3-brane describes a topologically stable domain wall,

an axion-type solution of the sine-Gordon equation in curved space-time. Analytical solutions for the domain wall profile and warp factor are presented. As expected, the 5D space turns out to be Anti-de-Sitter (AdS). Questions of compactification, Kaluza-Klein (KK) decomposition, graviphoton mass and other related issues are also discussed.

2 The Model

In this section we will consider higher dimensional ($D = 5$) gravity plus a complex scalar field which turns out to possess a non-factorizable solution of equation (2). The motivation for the choice of complex scalar is that with the help of Z_2 symmetry we naturally obtain a potential with a cosine profile [8] familiar from axion models. This yields a non trivial analytical solution for the θ -domain wall whose core can be identified as a 3-brane.

2.1 Complex Scalar Coupled to 5D Gravity

Consider 5D gravity coupled to a complex scalar field³ Φ through the action

$$S = \int d^5x \sqrt{G} \left(-\frac{1}{2} M^3 R - \Lambda + \mathcal{L}(\Phi) \right) , \quad (3)$$

with

$$\mathcal{L}(\Phi) = \frac{1}{2} G^{AB} (\partial_A \Phi^* \partial_B \Phi + \partial_B \Phi^* \partial_A \Phi) - V(\Phi) . \quad (4)$$

Here G_{AB} is the 5D metric tensor and $G = \text{Det} G_{AB}$ ($A, B = 1, \dots, 5$). The Einstein equation derived from (3) is given by

$$R_{AB} - \frac{1}{2} G_{AB} R - \frac{\Lambda}{M^3} G_{AB} = \frac{V}{M^3} G_{AB} + \frac{1}{M^3} (\partial_A \Phi^* \partial_B \Phi + \partial_B \Phi^* \partial_A \Phi) - \frac{1}{2M^3} G_{AB} G^{CD} (\partial_C \Phi^* \partial_D \Phi + \partial_D \Phi^* \partial_C \Phi) , \quad (5)$$

while the equation of motion for Φ follows from

$$\frac{\delta \mathcal{L}}{\delta \Phi} = \frac{1}{\sqrt{G}} \partial_A \left(\sqrt{G} \frac{\delta \mathcal{L}}{\delta (\partial_A \Phi)} \right) . \quad (6)$$

Terms on the right hand side of (5) effectively play the role of energy-momentum tensor T_{AB} , which will be the source for the dynamical generation of the 3-brane and yield a non-factorizable geometry.

³For higher dimensional non-factorizable scenarios, extended with real scalar fields, see [8]-[10].

Before proceeding to the specific model, which fixes $V(\Phi)$, let us derive the appropriate equations of motion [from (5), (6)]. We are looking for a metric of the form:

$$G_{AB} = \text{Diag}(A(y), -A(y), -A(y), -A(y), -1) , \quad (7)$$

which conserves 4D Poincare invariance:

$$ds^2 = A(y)\bar{g}_{\mu\nu}dx^\mu dx^\nu - dy^2 , \quad \bar{g}_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu} , \quad (8)$$

where

$$\eta_{\mu\nu} = \text{Diag}(1, -1, -1, -1) \quad (9)$$

and $\bar{h}_{\mu\nu}$ denotes the 4D graviton ($\mu, \nu = 1, \dots, 4$). The (1, 1) and (5, 5) components of (5) respectively give

$$\frac{A''}{A} = -\frac{2}{3} \frac{\Lambda + V}{M^3} - \frac{2}{3M^3} (\Phi^*)' \Phi' , \quad (10)$$

$$\left(\frac{A'}{A} \right)^2 = -\frac{2}{3} \frac{\Lambda + V}{M^3} + \frac{2}{3M^3} (\Phi^*)' \Phi' , \quad (11)$$

where primes denote derivatives with respect to the fifth coordinate y . Subtracting (11) and (10), we get:

$$-\frac{A''}{A} + \left(\frac{A'}{A} \right)^2 = \frac{4}{3M^3} (\Phi^*)' \Phi' . \quad (12)$$

Using the substitutions:

$$\Phi = v \cdot e^{i\theta} , \quad (13)$$

$$A = A_0 \cdot e^{-\sigma} , \quad (14)$$

and assuming that v in (13) does not depend on y [see discussion in sec. (1.2)], from (12) and (11) we derive:

$$\sigma'' = \frac{4v^2}{3M^3} \theta'^2 , \quad (15)$$

$$\sigma'^2 = -\frac{2}{3} \frac{\Lambda + V}{M^3} + \frac{2v^2}{3M^3} \theta'^2 . \quad (16)$$

With our assumption $v = \text{const.}$, from (6) we obtain the equation of motion for θ :

$$2v^2 \theta'' - 4v^2 \sigma' \theta' - \frac{\partial V}{\partial \theta} = 0 . \quad (17)$$

The three equations (15)-(17) are not independent. Namely, differentiating (16) and using (15), we obtain (17). However, in order for these three equations to have a solution, one fine tuning between the parameters is unavoidable. This can be seen from the following discussion: Solving equations (16) and (17) we have three parameters of integration $\theta(y_0)$, $\theta'(y_0)$ and $\sigma(y_0)$, where y_0 is some arbitrary point (In principle there is also a fourth parameter which expresses translation invariance. But it is irrelevant, since the equations are invariant under translations). $\sigma(y_0)$ is also irrelevant, since the equations contain only derivatives of σ . From (15), $\theta(y_0)$ is also irrelevant. Since for the brane solution we have to impose the condition $\theta'(\infty) = 0$, the third parameter $\theta'(y_0)$ is fixed from this condition. Therefore, there remains no free parameters, and for satisfying (15), one fine tuning must be done (for detailed discussions about this issue see [10]). This will be explicitly seen for the model discussed below.

2.2 Axionic Brane and Warped Geometry

We introduce a Z_2 symmetry under which $\Phi \rightarrow -\Phi$. The relevant potential is given by

$$V = \frac{\lambda_1}{4}(\Phi^* \Phi - v^2)^2 - \frac{\lambda}{2}(\Phi^2 + \Phi^{*2}) . \quad (18)$$

The first term in (18) is ($U(1)$) invariant under $\Phi \rightarrow e^{i\theta} \Phi$, while the last term explicitly breaks it to Z_2 . This avoids the appearance of a Goldstone mode because of non-zero Φ VEV⁴. We restrict our attention in (18) to terms needed to implement the scenario. The couplings $\Phi^4 + \Phi^{*4}$ can be included if so desired, but this makes analytic calculations more difficult. As noted in [12], such terms are absent in some models. Higher powers in Φ and Φ^* would complicate the discussion even further. We assume that the $U(1)$ violating term is such that

$$\lambda_1 v^2 \gg \lambda . \quad (19)$$

Therefore, the VEV $\langle |\Phi| \rangle$ is mainly determined by the first term in (18),

$$\langle \Phi^* \Phi \rangle \simeq v^2 , \quad (20)$$

from which

$$\Phi \simeq v e^{i\theta} . \quad (21)$$

⁴For models with Z_2 replacing the PQ symmetry and avoiding an undesirable axion, see papers [11, 12], where various phenomenological and cosmological implications are also studied.

Substituting (21) in (18), the θ dependent part of the potential is given by

$$V_\theta = -\lambda v^2 \cos 2\theta . \quad (22)$$

This type of potential was also used for brane formation in [8]. In our case we have obtained it through a Z_2 symmetry acting on a complex scalar field Φ . Assuming $\lambda > 0$, (22) acquires its minima for $\theta = 0, \pi$. The $\langle \theta \rangle$ VEV breaks the symmetry $\theta \rightarrow -\theta$. This causes the creation of topologically stable domain wall. The wall is stretched between two energetically degenerate minima, $\theta = 0$ and $\theta = \pi$. With assumption (19) it is consistent to consider v to be (essentially) y -independent.

Introducing the dimensionless coordinate ξ

$$\xi = \sqrt{2\lambda} y , \quad (23)$$

(15) and (17) respectively become:

$$2 \frac{\partial^2 \theta}{\partial \xi^2} - 4 \frac{\partial \sigma}{\partial \xi} \frac{\partial \theta}{\partial \xi} - \sin 2\theta = 0 , \quad (24)$$

$$\frac{\partial^2 \sigma}{\partial \xi^2} = \alpha \left(\frac{\partial \theta}{\partial \xi} \right)^2 , \quad (25)$$

where

$$\alpha = \frac{4v^2}{3M^3} . \quad (26)$$

Nontrivial solutions of (24) and (25), with boundary conditions

$$\theta(-\infty) = 0 , \quad \theta(+\infty) = \pi , \quad \sigma(\pm\infty) \propto \pm y , \quad (27)$$

[Note that due to the breaking of the $U(1)$ symmetry to Z_2 in (18), the wall here is not 'bounded by strings', a phenomenon encountered in $SO(10)$ and axion models [13].] will indicate the existence of 'warped' geometry and the axion(or θ)-brane (since $\langle \theta \rangle$ breaks 5D invariance). The point $\theta = \frac{\pi}{2}$ will be identified as the location of the 3-brane describing 4D theory.

Using the substitution

$$\theta = 2 \arctan f(\xi) , \quad (28)$$

(24), (25) can be rewritten as

$$-(f^2 - 1)f'' + 2f(f''f - f'^2) - 2\sigma'(f^2 + 1)f' + f(f^2 - 1) = 0 , \quad (29)$$

$$(f^2 + 1)^2 \sigma'' = 4\alpha f'^2 , \quad (30)$$

where primes denote derivatives with ξ . The form for f

$$f = ae^{m\xi} , \quad (31)$$

is a reasonable choice, where the parameters $a, m > 0$ are undetermined for the time being. Substituting (31) in (30), the latter can be integrated:

$$\sigma' = s_0 - 2\alpha m \frac{1}{1 + f^2} , \quad (32)$$

where s_0 is some constant. Substituting (32) into (29) and taking into account that $f'' = mf' = m^2 f$, we find:

$$-(f^2 - 1)m^2 - 2m[s_0(f^2 + 1) - 2\alpha m] + f^2 - 1 = 0 . \quad (33)$$

Comparing appropriate powers of f in (33), it is easy to verify that (33) is satisfied if

$$m = \frac{1}{\sqrt{1 + 2\alpha}} , \quad s_0 = \frac{\alpha}{\sqrt{1 + 2\alpha}} . \quad (34)$$

Integration of (32) gives

$$\sigma = \alpha \ln[\cosh(m\xi + \delta)] + \ln C , \quad \delta = \ln a \quad (35)$$

(C = constant and we have taken into account (34)).

Finally, for θ and the warp factor $A(= A_0 e^{-\sigma})$ we will have:

$$\theta = 2 \arctan(ae^{m\xi}) , \quad (36)$$

$$A = A_0 [\cosh(m\xi + \delta)]^{-\alpha} . \quad (37)$$

where the constant C is now absorbed in A_0 , and a still remains undetermined, which reflects translational invariance in the fifth direction ξ (y).

Let us note here that these solutions are obtained for $\lambda > 0$ and a negative sign in front of the last term in (18). In case of a positive sign, the potential is minimized for $\theta = \pm \frac{\pi}{2}$, and instead of the solution (36), we would have $\tilde{\theta} = \theta - \frac{\pi}{2}$. Indeed, under these modifications, equations (24), (25) are satisfied [for this case the sign in front of $\sin \tilde{\theta}$ in (24) will be positive, which reflects a change of sign of the last term in (18)].

From (37), taking into account (34), we will get the desirable asymptotic forms for A :

$$A \sim e^{s_0 \xi} , \quad \xi \rightarrow -\infty ,$$

$$A \sim e^{-s_0 \xi}, \quad \xi \rightarrow +\infty. \quad (38)$$

The solutions (35), (36) should also satisfy (16), which in terms of ξ has the form

$$\sigma'^2 = -\frac{\Lambda + V}{3\lambda M^3} + \frac{\alpha}{2}\theta'^2. \quad (39)$$

From (32), (36) and (34) we have

$$\sigma' = \alpha m \frac{f^2 - 1}{f^2 + 1}, \quad \theta' = \frac{2mf}{f^2 + 1}, \quad \cos 2\theta = 1 - \frac{8f^2}{(f^2 + 1)^2}. \quad (40)$$

Substituting all of this in (39), we can see that the latter is satisfied if

$$\Lambda = \lambda v^2(1 - 4\alpha m^2) = \lambda v^2 \frac{1 - 2\alpha}{1 + 2\alpha}. \quad (41)$$

Therefore, as we previously mentioned, one fine tuning between the parameters of the theory is necessary. The effective 5D cosmological constant is determined to be

$$\Lambda_{eff} = \Lambda + \langle V \rangle = \Lambda + V_\theta(\theta = 0, \pi) = -4\lambda v^2 \frac{\alpha}{1 + 2\alpha}. \quad (42)$$

As expected, the initial 5D space-time is AdS.

The warp factor (37) reaches its maximum at $\xi_0 = -\ln a/m$ and decays exponentially far from ξ_0 . For a realistic model which solves the gauge hierarchy problem, we may regard the axion wall as a hidden brane, located at ξ_0 . By placing the visible brane (which can describe our 4D Universe) at a distance $\Delta\xi \simeq 74/(\alpha m)$ from ξ_0 , all masses on the visible brane will be rescaled as $m_{\text{vis}} = M \cdot A(\xi_0 + \Delta\xi)^{1/2} \simeq M \cdot e^{-\alpha m \Delta\xi/2} \sim 10^{-16} \cdot M$. For $M \sim 10^{19}$ GeV, the desired scale $m_{\text{vis}} \sim \text{few} \cdot \text{TeV}$ will be naturally generated.

3 Kaluza-Klein Decomposition

In this section we will present the KK reduction of the 5D model to 4D. We will calculate the graviphoton $[(\mu, 5)$ component of metric tensor] mass, as well as the effective 4D Planck scale and the radius of the extra dimension. For KK reduction within models with non-factorizable geometry, also see papers in [14, 15], while for works addressing the effects of spontaneous breaking of higher Poincare invariance, Goldstone phenomenon and other relevant issues within models with non-warped geometry, see [16].

For performing a KK decomposition, it is convenient to rewrite the metric in (8) in a conformally 'flat' form:

$$ds^2 = \Omega^2(z) g_{MN} dx^M dx^N, \quad (43)$$

where:

$$dz = A^{-1/2}(y)dy , \quad \Omega^2(z) = A(y(z)) , \quad (44)$$

$$G_{MN} = \Omega^2 g_{MN} . \quad (45)$$

With the standard KK decomposition

$$g_{MN} = \begin{pmatrix} \bar{g}_{\mu\nu} - k^2 A_\mu A_\nu & k A_\mu \\ k A_\nu & -1 \end{pmatrix}, \quad g^{MN} = \begin{pmatrix} \bar{g}^{\mu\nu} & k A^\mu \\ k A^\nu & k^2 A_\alpha A^\alpha - 1 \end{pmatrix}, \quad (46)$$

where A_μ is the graviphoton, equation (43) reads

$$ds^2 = \Omega^2 \left(\bar{g}_{\mu\nu} dx^\mu dx^\nu - (dz + k A^\mu dx_\mu)^2 \right) . \quad (47)$$

We omit the graviscalar field in (46) since it is not relevant for our discussion. See [14] for a discussion involving this field. Eq. (47) acquires the ‘usual’ form for $A_\mu = 0$. For $A_\mu \neq 0$, it is invariant under the following transformations:

$$\begin{aligned} x'_\mu &= x_\mu , & z' &= z + \epsilon(x_\mu) , \\ A'_\mu &= A_\mu - \frac{1}{k} \partial_\mu \epsilon . \end{aligned} \quad (48)$$

Note that this is a $U(1)$ transformation for A_μ , where z plays the role of Goldstone field. The Z_2 symmetry breaking creates the brane and translational invariance in the fifth direction is spontaneously broken. The breaking of the corresponding generator gives rise to a massive A_μ field. By considering z as x_μ dependent (which corresponds to brane vibrations), the term $(\partial_\mu z)^2$ (see below) appear in the 4D action. This tell us that from the point of view of 4D observer, $z(x_\mu)$ is a Goldstone field which becomes the longitudinal component of A_μ . From this discussion it is clear that the fields $z(x_\mu)$ and A_μ reside on the 4D brane.

We now calculate the graviphoton mass. Taking into account (45), for the Einstein Tensor

$$\mathcal{G}_{MN} = R_{MN} - \frac{1}{2} G_{MN} R , \quad (49)$$

we have

$$\begin{aligned} \mathcal{G}_{MN}^G &= \mathcal{G}_{MN}^g + (D-2) (\nabla_M \ln \Omega \nabla_N \ln \Omega - \nabla_M \nabla_N \ln \Omega) + \\ & (D-2) g_{MN} \left(\nabla_P \nabla^P \ln \Omega + \frac{1}{2} (D-3) \nabla_P \ln \Omega \nabla^P \ln \Omega \right) , \end{aligned} \quad (50)$$

where \mathcal{G}^G and \mathcal{G}^g are calculated using G and g respectively. The covariant derivatives ∇_M are built from g , such that for a scalar function \mathcal{S}

$$\nabla_M \mathcal{S} = \partial_M \mathcal{S} , \quad (51)$$

while for a vector \mathcal{V}

$$\nabla_M \mathcal{V}^N = \partial_M \mathcal{V}^N + \Gamma_{MP}^N \mathcal{V}^P , \quad \nabla_M \mathcal{V}_N = \partial_M \mathcal{V}_N - \Gamma_{MN}^P \mathcal{V}_P . \quad (52)$$

From (49) we have

$$R = -\frac{2}{D-2} G^{MN} \mathcal{G}_{MN} , \quad (53)$$

and taking into account (50), we get:

$$\begin{aligned} R(G) &= \Omega^{-2} \left(R(g) - 2(D-1) \nabla_M \nabla^M \ln \Omega - (D^2 - 3D + 2) \nabla_M \ln \Omega \nabla^M \ln \Omega \right) = , \\ \Omega^{-2} &\left(R(g) - 2(D-1) \frac{\nabla_M \nabla^M \Omega}{\Omega} - (D^2 - 5D + 4) \frac{\nabla_M \Omega \nabla^M \Omega}{\Omega^2} \right) . \end{aligned} \quad (54)$$

Calculating $R(g)$ through (46) and keeping only relevant terms, we have

$$\begin{aligned} \sqrt{G} &= \sqrt{-\bar{g}} \Omega^5 , \\ R(g) &= \bar{R}(\bar{g}) + \frac{k^2}{4} F_{\mu\nu} F^{\mu\nu} + \dots , \end{aligned} \quad (55)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \quad (56)$$

and $\bar{R}(\bar{g})$ is the 4D curvature, built from the physical 4D metric $\bar{g}_{\mu\nu}$.

Taking into account (51), (52) and (46), we have:

$$\nabla_M \nabla^M \Omega = \left(\partial^\mu \partial_\mu \Omega + 2k A^\mu \partial_\mu \Omega' + (k^2 A^\mu A_\mu - 1) \Omega'' + \dots \right) , \quad (57)$$

$$\nabla_M \Omega \nabla^M \Omega = \left(\partial^\mu \Omega \partial_\mu \Omega + 2k \Omega' A^\mu \partial_\mu \Omega + (k^2 A^\mu A_\mu - 1) (\Omega')^2 \right) , \quad (58)$$

where primes here denote derivatives with respect to z . Using

$$\partial_\mu = \frac{\partial \bar{z}}{\partial x^\mu} \frac{\partial}{\partial \bar{z}} = \partial_\mu \bar{z} \cdot \frac{\partial}{\partial \bar{z}} , \quad \bar{z} \equiv z(x_\mu) , \quad (59)$$

from (54), (57) and (58) it finally follows that

$$R(G) = \Omega^{-2}R(g) - \Omega^{-2} \left(2(D-1)\frac{\Omega''}{\Omega} + (D^2 - 5D + 4)\frac{\Omega'^2}{\Omega^2} \right) \times \\ \left(\partial^\mu \bar{z} \partial_\mu \bar{z} + 2kA^\mu \partial_\mu \bar{z} + k^2 A^\mu A_\mu - 1 \right) . \quad (60)$$

From (60) we see that the field \bar{z} can be absorbed by A_μ by a suitable $U(1)$ transformation.

From the Einstein equation (5) we have:

$$-(\Lambda + V) = -\frac{M^3}{2} \frac{D-2}{D} R + \frac{1}{2} \frac{D-2}{D} G^{AB} (\partial_A \Phi^* \partial_B \Phi + \partial_B \Phi^* \partial_A \Phi) \quad (61)$$

and substituting this in (3), we get:

$$S = \int d^5x \sqrt{G} \left[-\frac{M^3}{2} \frac{2(D-1)}{D} R + \frac{2(D-1)}{D} \frac{1}{2} G^{AB} (\partial_A \Phi^* \partial_B \Phi + \partial_B \Phi^* \partial_A \Phi) \right] . \quad (62)$$

With

$$\frac{1}{2} G^{AB} (\partial_A \Phi^* \partial_B \Phi + \partial_B \Phi^* \partial_A \Phi) = \Omega^{-2} v^2 \theta'^2 \left(\partial^\mu \bar{z} \partial_\mu \bar{z} + 2kA^\mu \partial_\mu \bar{z} + k^2 A^\mu A_\mu - 1 \right) , \quad (63)$$

After integrating over the fifth dimension in (62), we obtain the reduced 4D action:

$$S^{(4)} = \int d^4x \sqrt{-\bar{g}} \left(-\frac{M_{\text{Pl}}^2}{2} \bar{R}(\bar{g}) - T - \frac{k^2}{4} B_{\mu\nu} B^{\mu\nu} + M_V^2 (B_\mu + \frac{1}{k} \partial_\mu \mathcal{Z}) (B^\mu + \frac{1}{k} \partial^\mu \mathcal{Z}) \right) , \quad (64)$$

where the 4D Planck mass is

$$M_{\text{Pl}}^2 = \frac{2(D-1)}{D} M^3 \int \Omega^3 dz , \quad (65)$$

the 4D brane tension is

$$T = M^3 \frac{D-1}{D} \int \Omega^3 \left[2(D-1) \frac{\Omega''}{\Omega} + (D^2 - 5D + 4) \frac{\Omega'^2}{\Omega^2} + \frac{2v^2}{M^3} \theta'^2 \right] dz \quad (66)$$

and the mass of the graviphoton is

$$M_V^2 = \frac{M^3}{M_{\text{Pl}}^2} \frac{D-1}{D} k^2 \int \Omega^3 \left[2(D-1) \frac{\Omega''}{\Omega} + (D^2 - 5D + 4) \frac{\Omega'^2}{\Omega^2} + \frac{2v^2}{M^3} \theta'^2 \right] dz . \quad (67)$$

In obtaining (64) we have used

$$B_\mu = M_{\text{Pl}} A_\mu , \quad B_{\mu\nu} = M_{\text{Pl}} F_{\mu\nu} , \quad \mathcal{Z} = M_{\text{Pl}} \bar{z} . \quad (68)$$

Comparing (66) and (67),

$$M_V^2 = \frac{T}{M_{\text{Pl}}^2} k^2 = \frac{T}{g_V^2 M_{\text{Pl}}^2} . \quad (69)$$

Simplifying (66) yields:

$$T = \frac{4}{5} M^3 \int dy A^2 \left(4 \frac{A_y''}{A} + \frac{A_y'^2}{A^2} + \frac{2v^2}{M^3} \theta_y'^2 \right) = \frac{16}{5} \sqrt{2\lambda} M^3 A_0^2 m \left(2\alpha^2 I(2\alpha) + (\alpha/2 - 2\alpha^2) I(2\alpha + 2) \right) , \quad (70)$$

where we have put $D = 5$, the subscript y denotes derivatives with respect to y , and

$$I(\alpha) = \int_0^1 \left(\frac{1 - \rho^2}{1 + \rho^2} \right)^\alpha \frac{d\rho}{1 - \rho^2} = \int_0^{\frac{\pi}{4}} (\cos 2t)^{\alpha-1} dt . \quad (71)$$

is some finite number whose value depends on the positive parameter α . For $\alpha = 1$, $I = \pi/4$, and for $\alpha = 2$, $I = 1/2$.

Note that the relation (69) between the graviphoton mass and brane tension, has same form as for models with non warped geometry [16].

Simplifying (65) one finds:

$$M_{\text{Pl}}^2 = \frac{8}{5} M^3 \int_{-\infty}^{+\infty} A(y) dy = M^3 R_{eff} , \quad (72)$$

where

$$R_{eff} = \frac{8}{5\sqrt{2\lambda}} \int_{-\infty}^{+\infty} A(\xi) d\xi = \frac{8A_0}{5\sqrt{2\lambda}} \int_{-\infty}^{+\infty} [\cosh(m\xi + \delta)]^{-\alpha} d\xi = \frac{32A_0}{5m\sqrt{2\lambda}} I(\alpha) . \quad (73)$$

Thus, even though the extra dimension y is non-compact, its ‘effective’ size R_{eff} is finite. In this sense the extra space is effectively compact. Expression (72) resembles the well known relation $M_{\text{Pl}}^2 \sim M^{2+n} L^n$ (for $n = 1$), which relates the effective 4D Planck scale to the fundamental scale M and the volume ($\sim L^n$) of the n extra dimensions [1, 2]. The crucial difference from models [1, 2] is that even for values $M \sim M_{\text{Pl}}$, $R_{eff} \sim 1/M_{\text{Pl}}$ in (72), the desired hierarchy is obtained, thanks to the warped geometry.

In conclusion, it would be interesting to investigate the possibility of introducing a second domain wall, located at a suitable distance from the first and characterized by the

TeV scale . 'Double wall' solutions that are dynamically stabilized in axion type models with Minkowski background have been studied in ref. [12]. Some extension of the model considered here may well be required to implement such a scenario.

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